Quantum Zeno Effect, Instability, and Decay

Quantum Dynamics, encompassing all change in a Quantum system, is formulated in terms of states and their changes in time. In most cases, this evolution is given by either the Schroëdinger equation, or its equivalent in Heisenberg formulation. In addition to motion, it also describes transitions like the de-excitation of an atom accompanied by the emission of a photon, and like the photoelectric effect in which a photon is absorbed and an electron is liberated. Among such transitions is decay in which the initial state of a system is changed into what is apparently another system. In this case we talk of a transition or decay.

In a decay, the initial (discrete) state is diminished and there is a compensating increase in the final state. This is most evident in radioactive decays, in which an initial nucleus gives birth to a final nucleus and some alpha or beta particles. The classical law tells us that the rate of survival of the parent nucleus must be exponential with a characteristic time:

$$N(t) = N(0)e^{-t/\tau} \tag{1}$$

The only assumption is that the probability of decay per unit time is proportional to the existing number: $dN(t)/dt = -N(t)/\tau$.

In some sense, this law must also exist in Quantum theory. In Quantum theory we can obtain the survival probability $P(t)$ as the square of a survival amplitude $A(t)$. The latter is defined as the overlap of the evolved state at time t with the initial state. By a straightforward calculation it could be rewritten as the Fourier transform of the energy spectrum $\sigma(\lambda)$

$$A(t) = \int_0^\infty e^{-i\lambda t}\sigma(\lambda)d\lambda \tag{2}$$

The lower limit being 0 incorporates the essential requirement that the energy is bounded below. It is clear from this expression that we cannot ever have a strictly exponential decay. The latter would imply that the energy spectrum goes from $-\infty$ to $+\infty$. How does this form affect the decay probability?

Since the expression for $A(t)$ is analytic for complex time in the upper half plane, using the Paley-Weiner theorem Khalfin [1] showed that there could not be a strictly exponential survival amplitude for large times. But what about small times?

Since the survival amplitude should give $A(t) = 1 - i\lambda t + \mathcal{O}(\lambda^2 t^2)$ for small enough time,

the survival probability is given by

$$P(t) = |A(t)|^2 = 1 + \mathcal{O}(\lambda^2 t^2). \tag{3}$$

So, for small enough times, the decay rate which is the derivative of the decay probability is proportional to the time and goes to zero for time going to zero. This is in direct contrast with the radioactive law which postulates a constant rate at all times.

As a consequence, if an unstable system is observed very frequently (and reset at each time it is observed), the survival amplitude remains near unity.

$$P(t) \rightarrow \left[P\left(\frac{t}{N}\right) \right]^N = \left[1 - \left(\frac{t}{N}\right)^2 - \dots \right]^2. \tag{4}$$

In the limit of N increasing with t kept constant, we get $P(t) \rightarrow 1$.

This is the Quantum Zeno Effect formulated by Misra and Sudarshan [2]. The same study of survival amplitudes can be extended to uninterrupted decay to find the detailed law of survival. A paper written by Chiu and Sudarshan [3] shows that there are several time regions; the Zeno region for very small times, the Khalfin region for very long times, and the intermediate time region. For many of the models studied, the middle region is experimentally accessible. The detailed behavior depends on the analytic behavior of the propagator. If there is a discrete pole in the lower half plane of the "second sheet", it will contribute to a near exponential decay [4]. But from general principles of Quantum Mechanics, one can conclude that such poles must be accompanied by suitable branch cuts which modify the detailed time evolution [5–8].

Quantum Zeno Effect obtains when a metastable Quantum state is measured repeatedly and reset sufficiently often. Aharanov (and collaborators) extended this result by showing that if the successive measurements are associated with the projection to a sequence of states which have a prescribed time dependence, we could have the Quantum state actually pass through this sequence of states. (It is as if we can gently lead the Quantum state along any desired path !). [9]

Since small time scales are not available to experiments, the verification of Quantum Zeno Effect requires some ingenuity. The effect was first empirically verified by Valanju, Chiu, and Sudarshan [10]. They studied the multiplicity of pions produced in high energy cosmic ray proton collisions with large nuclei. When a proton collides with a nucleus of atomic mass M we expect effectively $M^{1/3}$ collisions and hence $M^{1/3}$ times the multiplicity in a single

nucleon-nucleon collision. In their analysis they found that the multiplicity was much less. They quantitatively explained this as resulting from frequent "observations" by the target nucleons on the colliding nucleon in times of the order of the internucleon distance (~ 1 Fermi) divided by the speed of light.

The first direct experiment to test the Quantum Zeno Effect was carried out by Itano and collaborators at NIST and at the University of Colorado [11]. They studied transitions from the excited state e to ground state g in an ion with three levels. The third level, i, is an intermediate meta-stable state. In their experiment, the electron is initially in state g or e. Using a laser, the electron can be excited from g to i. A by-product of this excitation is the emission of a fluorescent photon when the electron makes the transition from i to g. These fluorescent photons were recorded by their detectors. The excited level e is connected to level i only through the level g. If a series of photons are incident upon the atom, the electron will make several transitions from g to i and back to g, emitting one fluorescent photon per cycle. But now, if the electron initially is in state e then the photon will have no electron in the state g to interact with; so nothing will happen (i.e. no emission of fluorescent photon by the ion). The lack of emission from the atom can be interpreted as indirect "observation" of the occupied state e. Their experiment showed that the life-time of e was extended as the rate of incident photons became higher (large time period without fluorescent photons), hence the Quantum Zeno Effect.

More recently Mark Raizen and collaborators at The University of Texas at Austin [12] studied Sodium atoms trapped in a magneto-optical trap. They studied the tunneling of atoms from the trap, and found that when atom population in the trap was "observed" frequently, fewer atoms escaped the trap. This was the first observation of the Quantum Zeno Effect in an unstable system. Sudarshan and his students Modi and Shaji theoretically modeled and reproduced all the results of this Quantum Zeno experiment [13].

Expanding on these ideas, Sudarshan has recently developed a generalization of the usual quantum theory into the complex energy plane, and operators with a complex spectrum, yielding Quantum Dynamics in Dual Spaces [14]. With this theory he has calculated that the decay of the neutral kaon leads to a very small but unmistakable correction, which is often interpreted as part of the time reversal violation in the kaon decay [15].

The importance of the Quantum Zeno Effect can no longer be denied. Literature is flooded by this topic for it has a pivotal place in understanding quantum theory of measurement

and is a vital tool in the field of quantum computing. Quantum Zeno Effect is also a strong candidate in the fight against decoherence, which has been the most crippling challenge of mother nature when it comes to storing a quantum state.

---

[1] L. A. Khalfin, Zh. Eksp. Teor. Fiz. 33, 1371 (1957) [Sov. Phys. JETP 6, 1053 (1958)].

[2] "The Zeno's Paradox in Quantum Theory"; with B. Misra, J. Math Phys. 18(4) 756-763 (1977).

[3] "Time Evolution of Unstable Quantum States and a Resolution of Zeno's Paradox"; with C. B. Chiu and B. Misra, Phys. Rev. D 16, 520 (1977).

[4] "Decaying States as Complex Energy Eigen Vectors in Generalised Quantum Mechanics"; with C. B. Chiu and V. Gorini, Phys. Rev. D18, 2914 (1978).

[5] "Decay and Evolution of the Neutral Kaon"; with C. B. Chiu, Phys. Rev. D42, 3712 (1990).

[6] "Unstable Systems in Generalised Quantum Theory", with Charles B Chiu and G. Bhamathi, Advances in Chemical Physics XCIX, John Wiley and Sons, Inc. (1997), pp.121-210.

[7] "Quantum Zeno Dynamics"; with P. Facchi, V. Gorini, G. Marmo, S. Pascazio, Phys. Lett. A 275, 12 (2000).

[8] "Zeno Dynamics with Constraints"; with P. Facchi, G. Marmo, S. Pascazio and A. Scardicchio, J. Optics B: Quantum Optics 6, S492 (2004).

[9] "Meaning of an individual 'Feynman path' ", Y. Aharanov and M. Vardi, Phys. Rev. D 21, 8, 2235 (1980).

[10] "Spatio-temporal Development of Hadron-Nucleus Collisions"; with P. Valanju and C. B. Chiu, Phys. Rev. D 21, 1304 (1980), and "Spatio-temporal Development of Hadron-Nucleus Collisions"; P. Valanju, Ph.D thesis, Univ. of Texas at Austin, (1980).

[11] "Quantum Zeno effect", Wayne M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland, Phys. Rev. A 41, 22952300 (1990).

[12] "Observation of the Quantum Zeno and Anti-Zeno Effects in an Unstable System", M. C. Fischer, B. Gutirrez-Medina, and M. G. Raizen, Phys. Rev. Lett. 87, 040402 (2001).

[13] "The quantum Zeno effect: A solvable model for indirect pre-measurement", with Modi and Shaji, J. Phys. A: Math. Gen. 37, 46, 11285 (2004), and "Quantum Zeno and anti-Zeno effects in two decaying discrete states", K. Modi and A. Shaji, arXiv:quant-ph/0502075 (2004).

[14] "Quantum Dynamics in Dual spaces" Phys. Rev. A 50, 2006 (1994).

[15] "Theory of the neutral kaon system", With C. B. Chiu, in "A gift of prophecy", E.C.G. Sudarshan (ed.) World Scientific (1995) p. 81.

Additional references:

[16] "The Time Scale for Quantum Zeno Paradox and Proton Decay"; with B. Misra and C. B. Chiu, Phys. Lett. B 117, 34 (1982).

[17] "Natural Law", Proceedings of 1983 Theoretical Physics Meeting at Amalfi (May 6-7, 1983), Dell'Universita Degli Studi Di Salerno, Italy (1984).