Symmetry

Newton's First and Second Laws of Motion guarantee momentum conservation laws. Displacement invariance is at the root of these conservation laws. Similarly, if the dynamical laws are independent of time translation and of orientation in the three space dimensions, it leads to the conservation of energy and of angular momentum. Nöether gave the general relation between invariance and conservation laws. Such principles enable us to solve collision problems even when we do not know the detailed dynamical law.

This set of invariances could be enlarged by the transformation to moving coordinate frames. This leads to the Galilei group or the Lorentz group depending on the law of addition of velocities [1]

Apart from these space-time symmetries, we have "internal" symmetries. Some of them are approximate (or "broken"). Heisenberg proposed that the nucleons, protons and neutrons, exhibit a symmetry which demands the (approximate) charge independence of the nucleon–nucleon interaction. When meson theory of nuclear forces was considered soon after the charged pions were discovered, Kemmer showed that the charge independence of nuclear forces demands the existence of a neutral pion. He further showed that this structure is invariant under the isospin $SU(2)$ group. Hence the coupling strength of neutron-proton-charged meson is twice that of the coupling strength of neutral pion to the proton and neutron. Charge independence demands specific relations between the couplings:

$$g_{\pi^+np} = g_{\pi^-pn} = \sqrt{2}g_{\pi^0pp} = -\sqrt{2}g_{\pi^0nn}. \tag{1}$$

For any linear electromagnetic property like anomalous magnetic moment of the charge distribution which depends linearly on the third component of the isospin, sum rules can be obtained. For example, if μ_+, μ_0, μ_- are the magnetic moments of the isospin triplet Σ_+, Σ_0, Σ_-, they must be of the form

$$\mu_+ = a + b, \quad \mu_0 = a, \quad \mu_- = a - b \tag{2}$$

for appropriate a and b. Hence, they satisfy the sum rule [2]

$$\mu_+ + \mu_- = 2\mu_0 \tag{3}$$

For quadratic properties of these particles, we have an expression of the second degree in the third component of the isospin. For example, for the Δ resonance, we get

$$m_{++} = \alpha + \frac{3}{2}\beta + \frac{9}{4}\gamma \tag{4}$$

$$m_{+} = \alpha + \frac{1}{2}\beta + \frac{1}{4}\gamma \tag{5}$$

$$m_{0} = \alpha - \frac{1}{2}\beta + \frac{1}{4}\gamma \tag{6}$$

$$m_{-} = \alpha - \frac{3}{2}\beta + \frac{9}{4}\gamma \tag{7}$$

Eliminating α, β, and γ, we get the sum rule [3]

$$(m_{++} - m_{-}) = 3(m_{+} - m_{0}) \tag{8}$$

These obtain by the application of the Wigner-Eckert theorem to isospin multiplets. Similar considerations apply to the hadronic weak interaction currents [4].

When it comes to the broken $SU(3)$ symmetry of hadrons, we can obtain similar results for the electromagnetic properties of $SU(3)$ multiplets. The basic octet relations for the transition magnetic moment from Σ^0 to Λ^0 are [3]:

$$\mu_{\Sigma^0-\Lambda^0} = \frac{\sqrt{3}}{2}\left(\mu_{\Lambda^0} - \mu_{\Sigma^0}\right). \tag{9}$$

The electromagnetic mass differences satisfy two sum rules

$$M(p) - M(n) + M(\Xi^0) - M(\Xi^-) + M(\Sigma^-) - M(\Sigma^+) = 0 \tag{10}$$

and the transition mass is

$$\sqrt{3}M_{+}(\Sigma^0, \Lambda) = M(n) - M(p) - M(\Sigma^0) + M(\Sigma^+). \tag{11}$$

For the baryon resonance decimet (10 fold representation) we obtain simpler formulae [5].

Soon after the work of Kemmer, who established the coupling scheme for pions and nucleons, Heitler established a linear relation between the four observable cross-sections of the processes $\pi^+p \to \pi^+p$, $\pi^-p \to \pi^-p$, $\pi^-p \to \pi^0n$, and $\pi^-n \to \pi^-n$. These relations may be obtained by adding the two complex isospin amplitudes for $I = \frac{3}{2}$ and $I = \frac{1}{2}$. Therefore there are three unknown quantities and four cross-sections and one relation must obtain between them. This is Heitler's relation.

The kaon decays into two pions containing three amplitudes, $K^0 \to \pi^+\pi^-$, $K^0 \to \pi^0\pi^0$ and $K^+ \to \pi^+\pi^0$ but charge independence establishes a sum rule connecting these three transition rates [6].

But when higher groups like $SU(3)$ and $SU(6)$ are involved, the computation of the various amplitudes is more tedious. There is a simple arithmetical method to construct such relations. This is Shmushkevich's method for $SU(2)$ [7]. Sudarshan and collaborators discovered the rationale of this method and traced it back to Schur's lemma [8]. The inverse problem of looking for the origin of symmetry was studied by Sudarshan, O'Raifeartaigh, and Santhanam [9]. For a generic system, we consider the initial particles α, r and final particles β, s. The quantity

$$\sum_{\alpha=\beta} F_{\alpha r,\beta s} = G_{rs} \tag{12}$$

is invariant under the group since $F_{\alpha r,\beta s}$ itself is an invariant. But G_{rs} is an invariant second rank mixed tensor and by Schur's lemma it is a multiple of the Kronecker delta in r and s. This elegant proof is applicable to any group if $\alpha, r; \beta, s$ belong to irreducible representations. Sudarshan and collaborators have given explicit results for $SU(2)$ [7].

There is the additional symmetry of particles and antiparticles. From the combined strong reflection TCP we may conclude that the masses and lifetimes (if the particles are unstable) are strictly equal (though the partial decay widths may not be equal). If the interchange of all species of particles and their antiparticles leaves the physical amplitudes unchanged, we say that charge conjugation invariance holds. Both the strong and the electromagnetic interactions are invariant under charge conjugation. Then we can deduce selection rules. For electromagnetism, this result is Furry's theorem, which states that a state of an even number of photons cannot ever transform into an odd number of photons if there are no other particles present. An electron-positron state may decay only into two or three photons, depending upon the orbital and spin angular momenta. Combining the charge conjugation invariance with the isospin invariance, one can get new selection rules; these were discovered by Michel [10] and by Lee and Yang [11].

The problem of combining internal symmetry with the Lorentz group was studied by Sudarshan and collaborators [12].

In addition to the exact coupling coefficients of baryons and mesons, one can obtain relations between them if the symmetry breaking transforms as an octet.

Many of the results of $SU(3)$ may be obtained by looking at the discrete subgroup of the Weyl reflections, as was done for the decay widths of hadrons. These Weyl reflections degenerate into the charge symmetry transformation for $SU(2)$ symmetry [5].

When one considers time reversal, the quantum mechanical transformation is anti-linear. But the time reversed state of a charged particle is the charged particle itself. The process of time reversal on $SU(3)$ (or higher group) multiplets (explicitly studied by Sudarshan and Biedenharn) demonstrates the "coordinate independent" form of time reversal [13].

In the realm of space-time behavior, the consequences of Lorentz invariance for classical (or quantum) particles may be studied. If we demand a Hamiltonian formulation of the interacting system and include the explicit "world line condition" then there can be no interaction [14, 15]. This "No-Go theorem" originally deduced for two particles in a Hamiltonian formulation can be adapted to any number of interacting particles and equally well for an arbitrary number of particles [16, 17].

How do we then describe interaction of relativistic particles? A formalism for this was given by Komar's explicitly covariant relations involving four-momenta and four-coordinates for each particle together with invariant constraints. Sudarshan and Mukunda have shown that several versions of this are possible [15]. In these considerations, time emerges as a realization of the constraints [14].

While most of the groups of internal symmetries are compact, Galilei (or Lorentz) group is not. One can also study the group representation for non-compact groups. Sudarshan, Mukunda, and Kuriyan developed a general method (Master Analytic Representation) of finding all irreducible representations of non-compact groups including both the principal series and the auxiliary series [18]. The examples of $O(4,1)$ and $O(5,1)$ have been worked out. Sudarshan and Kuriyan used group theory to find relations between meson-baryon scattering amplitudes in strong and intermediate coupling regimes [19].

A system of two qubits exhibits the proprety of entanglement originally introduced by Schröedinger in the context of the Einstein-Podolsky-Rosen paradox of distant correlations. Such entanglement is a resource for quantum computing. The question of the relativistic behaviour of entanglement asks whether sub-systems entangled in one frame are entangled in another frame. Jordan, Shaji, and Sudarshan proved that entanglement is not relativistically invariant, contradicting previous published claims [20].

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